

Name: _____

Date: _____

SUMMATION NOTATION COMMON CORE ALGEBRA II



Much of our work in this unit will concern **adding the terms of a sequence**. In order to specify this addition or summarize it, we introduce a new notation, known as **summation or sigma notation** that will represent these sums. This notation will also be used later in the course when we want to write formulas used in statistics.

SUMMATION (SIGMA) NOTATION

$$\sum_{i=a}^n f(i) = f(a) + f(a+1) + f(a+2) + \cdots + f(n)$$

where ***i*** is called the **index variable**, which starts at a value of ***a***, ends at a value of ***n***, and moves by unit increments (increase by 1 each time).

Exercise #1: Evaluate each of the following sums.

(a) $\sum_{i=3}^5 2i$

(b) $\sum_{k=-1}^3 k^2$

(c) $\sum_{j=-2}^2 2^j$

(d) $\sum_{i=1}^5 (-1)^i$

(e) $\sum_{k=0}^2 (2k+1)$

(f) $\sum_{i=1}^3 i(i+1)$

Exercise #2: Which of represents the value of $\sum_{i=1}^4 \frac{1}{i}$?

(1) $\frac{1}{10}$

(3) $\frac{25}{12}$

(2) $\frac{9}{4}$

(4) $\frac{31}{24}$



Exercise #3: Consider the sequence defined recursively by $a_n = a_{n-1} + 2a_{n-2}$ and $a_1 = 0$ and $a_2 = 1$. Find the value of $\sum_{i=4}^7 a_i$

It is also good to be able to place sums into sigma notation. These answers, though, will not be unique.

Exercise #4: Express each sum using sigma notation. Use i as your index variable. First, consider any patterns you notice amongst the terms involved in the sum. Then, work to put these patterns into a formula and sum.

(a) $9 + 16 + 25 + \cdots + 100$

(b) $-6 + -3 + 0 + 3 + \cdots + 15$

(c) $\frac{1}{25} + \frac{1}{5} + 1 + 5 + \cdots + 625$

Exercise #5: Which of the following represents the sum $3 + 6 + 12 + 24 + 48$?

(1) $\sum_{i=1}^5 3^i$

(3) $\sum_{i=0}^4 6^{i-1}$

(2) $\sum_{i=0}^4 3(2)^i$

(4) $\sum_{i=3}^{48} i$

Exercise #6: Some sums are more interesting than others. Determine the value of $\sum_{i=1}^{99} \left(\frac{1}{i} - \frac{1}{i+1} \right)$. Show your reasoning. This is known as a **telescoping series (or sum)**.



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SUMMATION NOTATION

COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Evaluate each of the following. Place any non-integer answer in simplest rational form.

(a) $\sum_{i=2}^5 4i$

(b) $\sum_{k=0}^3 (k^2 + 1)$

(c) $\sum_{j=-2}^0 (2j + 1)$

(d) $\sum_{i=-1}^3 2^i$

(e) $\sum_{k=0}^2 (-1)^{2k+1}$

(f) $\sum_{i=1}^3 \log(10^i)$

(g) $\sum_{n=1}^4 \frac{n}{n+1}$

(h) $\frac{\sum_{i=2}^4 (i+1)^2}{\sum_{i=2}^4 (i^2 + 1)}$

(i) $\sum_{k=0}^3 256^{\frac{1}{2^k}}$

2. Which of the following is the value of $\sum_{k=0}^4 (4k + 1)$?

(1) 53

(3) 37

(2) 45

(4) 80

3. The sum $\sum_{i=4}^7 2^{i-7}$ is equal to

(1) $\frac{15}{8}$

(3) $\frac{3}{4}$

(2) $\frac{3}{2}$

(4) $\frac{7}{8}$



4. Write each of the following sums using sigma notation. Use k as your index variable. Note, there are many correct ways to write each sum (and even more incorrect ways).

(a) $-2 + 4 + -8 + \cdots + 64 + -128$ (b) $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{81} + \frac{1}{100}$ (c) $4 + 9 + 14 + \cdots + 44 + 49$

5. Which of the following represents the sum $2 + 5 + 10 + \cdots + 82 + 101$?

(1) $\sum_{j=1}^6 (4j - 3)$ (3) $\sum_{j=1}^{10} (j^2 + 1)$
 (2) $\sum_{j=3}^{103} (j - 2)$ (4) $\sum_{j=0}^{11} (4^j + 1)$

6. A sequence is defined recursively by the formula $b_n = 4b_{n-1} - 2b_{n-2}$ with $b_1 = 1$ and $b_2 = 3$. What is the value of $\sum_{i=3}^5 b_i$? Show the work that leads to your answer.

REASONING

6. A curious pattern occurs when we look at the behavior of the sum $\sum_{k=1}^n (2k - 1)$.

- (a) Find the value of this sum for a variety of values of n below:

$$n = 2: \sum_{k=1}^2 (2k - 1) =$$

$$n = 4: \sum_{k=1}^4 (2k - 1) =$$

$$n = 3: \sum_{k=1}^3 (2k - 1) =$$

$$n = 5: \sum_{k=1}^5 (2k - 1) =$$

- (b) What types of numbers are you summing?
What types of numbers are the sums?

- (c) Find the value of n such that $\sum_{k=1}^n (2k - 1) = 196$.

